Microwave Engineering

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SCOPES:

Passive microwave circuits design and analysis using transmission line theory and microwave network theory.

***TEXTBOOK AND REFERENCE BOOKS*:**

1. **“Microwave Engineering”, by David M. Pozar, 3rd Edition (Textbook)**
2. “Foundations for Microwave Engineering”, by R. E. Collin, 2ed Edition (Ref)
3. “Microwave Engineering”, by Peter A. Rizzi, (Out of Print) (Ref).

Outline

1. Transmission Line Theory
2. Transmission Lines and Waveguides

   General Solutions for TEM, TE, and TM waves ; Parallel Plate waveguide ; Rectangular Waveguide ; Coaxial Line ; Stripline ; Microstrip

3. Microwave Network Analysis

   Impedance and Equivalent Voltages and Currents ; Impedance and Admittance Matrices ; The Scattering Matrix ; ABCD Matrix ; Signal Flow Graphs ; Discontinuties and Model Analysis

4. Impedance Matching and Tuning

   Matching with Lumped Elements ; Single-Stub Tuning ; Double-Stub Tuning ; The Quarter-Wave Transformer ; The Theory of Small Reflections

5. Microwave Resonators

   Series and Parallel Resonant Circuits ; Transmission Line Resonators ; Rectangular Waveguide Cavities Dielectric Resonators

6. Power Dividers and Directional Couplers

   Basic Properties of Dividers and Couplers ; The T-Junction Power Divider ; The Wilkinson Power Divider ;

   Coupled Line Directional Couplers ; 180^o hybrid

7. Microwave Filters

   Periodic Structure ; Filter Design by the Insertion Loss Method ; Filter Transformations ; Filter Implementation ;

Introduction

*Definition

Microwave: designating or of that part of the electromagnetic spectrum between the far infrared and some lower frequency limit: commonly regarded as extending from 300,000 to 300 megahertz. (from Webster's dictionary)

f : 300MHz - 300GHz ⇨ λ : 100cm - 0.1cm

electromagnetic spectrum

*Why use microwaves

(1) Antenna gain is proportional to the electric size of the antenna.

⇨ f ↑, gain ↑

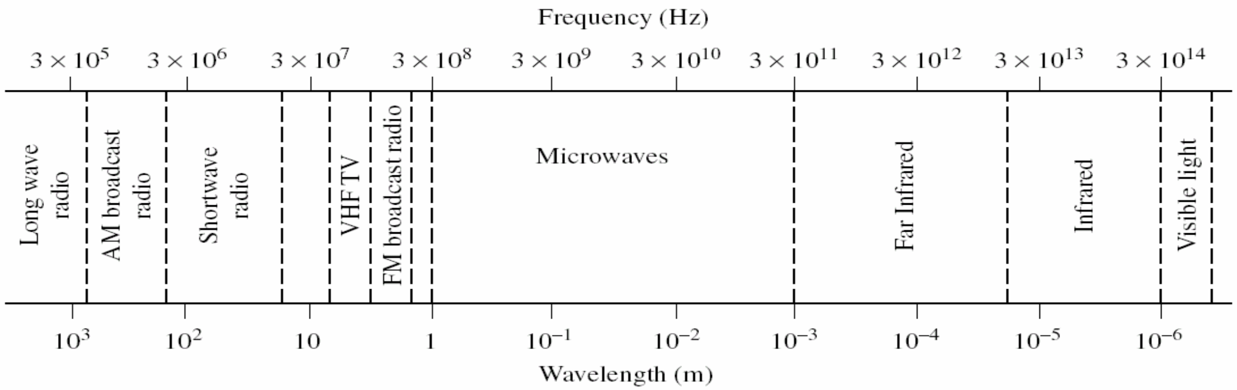
⇨ miniature microwave system possible

(2) f ↑ ⇨ available bandwidth ↑

e.g., TV BW=6MHz

10% BW of VHF @60MHz for 1channel

1% BW of U-band @60GHz for 100 channels



Typical Frequencies	
AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz
	869–894 MHz
European GSM cellular	880–915 MHz
	925–960 MHz
GPS	1575.42 MHz
	1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz
	2.400–2.484 GHz
	5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

Approximate Band Designations	
Medium frequency	300 kHz to 3 MHz
High frequency (HF)	3 MHz to 30 MHz
Very high frequency (VHF)	30 MHz to 300 MHz
Ultra high frequency (UHF)	300 MHz to 3 GHz
L band	1–2 GHz
S band	2–4 GHz
C band	4–8 GHz
X band	8–12 GHz
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz
U band	40–60 GHz
V band	50-75 GHz
E band	60-90 GHz
W band	75-110 GHz
F band	90-140 GHz

The Electromagnetic Spectrum

(3) Line of sight propagation and not effected by cloud, fog,…

→ frequency reuse in satellite and terrestrial communications

(4) Radar cross section (RCS) is proportional to the target electrical size.

→ frequency ↗, RCS ↗

→ radar application

(5) Molecular, atomic and nuclear resonances occur at microwave frequencies

→ astronomy, medical diagnostics and treatment, remote sensing and industrial heating applications

*Biological effects and safety

non-ionized radiation → thermal effect

IEEE standard C95.1-1991

Excessive radiation may be dangerous to brain, eye, genital, …..

→ cataract, sterility, cancer,…..

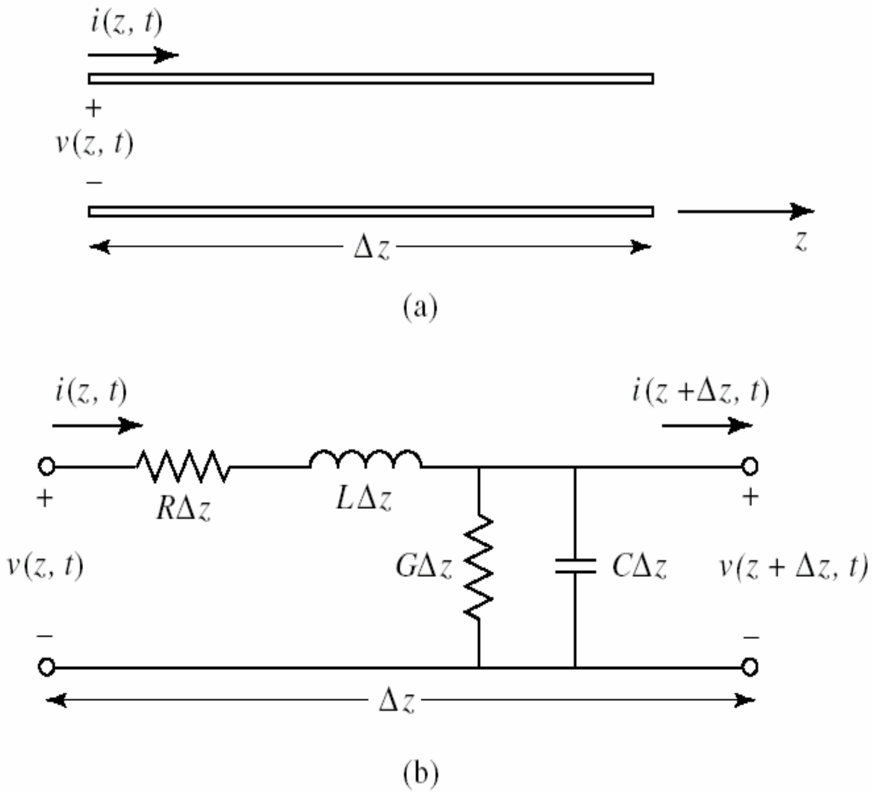
1. Transmission Line Theory

The Lumped-Element Circuit Model for a Transmission Line
The Terminated Lossless Transmission Line
Smith Chart
Quarter-Wave Transformer
Generator and Load Mismatched
Lossy Transmission Lines

The Lumped-Element Circuit Model for A Transmission Line

A transmission line is a distributed-parameter network, where voltages and currents can vary in magnitude and phase over its length.

A transmission line is often schematically represented as a two-wire line, since transmission line => TEM wave propagation



: coaxial line, parallel line and stripline

Lumped-Element Circuit Model:

R = series resistance per unit length (both conductors)

L = series inductor per unit length (both conductors)

G = shunt conductance per unit length

C = shunt capacitance per unit length

From Kirchhoff's voltage and Kirchhoff's current law

KVL, KCL $\Rightarrow$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

$$\frac{\partial i(z,t)}{\partial z} = -Gi(z,t) - C\frac{\partial v(z,t)}{\partial t}$$

time-domain transmission line, or telegrapher equation

$\Rightarrow$ time-harmonic form

$$\frac{dV(z)}{dz} = -(R + jwL)I(z)$$

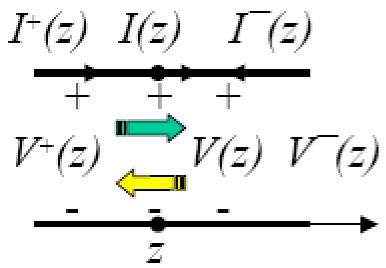
$$\frac{dI(z)}{dz} = -(G + jwC)V(z)$$

$\Rightarrow$ wave equation

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0, \qquad \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0$$

$$\gamma \equiv \sqrt{(R + jwL)(G + jwC)} \equiv \alpha + j\beta \quad \text{propagation constant}$$

- Traveling wave solutions



$$V(z) = V^+(z) + V^-(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z}$$

$$I(z) = I^+(z) + I^-(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z} = \frac{V_o^+}{Z_o} e^{-\gamma z} - \frac{V_o^-}{Z_o} e^{\gamma z}$$

$$\Rightarrow Z_o \equiv \sqrt{\frac{R + jwL}{G + jwC}} = \frac{V_o^+}{I_o^+} = -\frac{V_o^-}{I_o^-}$$ characteri stic impedance

where $I(Z) = \dfrac{\gamma}{R + j\omega L}\left[V_o^+ e^{-\gamma z} - V_o^- e^{\gamma z}\right]$

$$Z_o = \frac{R + j\omega L}{\gamma}$$

time - domain solution

$$v(z,t) = |V_o^+| e^{-\alpha z} \cos(wt - \beta z + \angle V_o^+) + |V_o^-| e^{\alpha z} \cos(wt + \beta z + \angle V_o^-)$$

$$i(z,t) = |I_o^+| e^{-\alpha z} \cos(wt - \beta z + \angle I_o^+) + |I_o^-| e^{\alpha z} \cos(wt + \beta z + \angle I_o^-)$$

where phase constant $\beta = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{v_p}$

phase velocity $v_p = \dfrac{\omega}{\beta} = \lambda f$

input impedance $Z_{in}(z) = \dfrac{V(z)}{I(z)}$

For Lossless Line

$$From\ previous\ drived\ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad ; \quad Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}\ ; R\ and\ G\ are\ loss$$

$$if\ let\ R\ and\ G\ are\ zero:$$

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC} \quad (\alpha = 0\ \beta = \omega\sqrt{LC}); Z_0 = \sqrt{\frac{L}{C}}$$

$$from\ \ v_{pn} = \frac{\omega}{\beta}\ \ to\ obtained\ v_{pn} = \frac{1}{\sqrt{LC}} \quad if\ dielectric\ medium\ is\ air: v_{pn} = c$$

The Terminated Lossless Transmission Line

Assume that an incident wave of the form $V_o^+ e^{-j\beta z}$ *is generated from a source at* $z < 0$. → *we have seen that the ratio of voltage to current for such a traveling wave is* Z_o, *characteristic impedance.*

→*When the line is terminated in an arbitrary load* $Z_L \neq Z_o$, *the ratio of voltage to current at the load must be* Z_L; *a reflected wave must be excited with the appropriate amplitude to satisfy this condition.*

Sum of incident and reflected waves standing wave solution

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$$

The total voltage and current at the load are related by the load impedance

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_o \qquad at\ z = 0$$

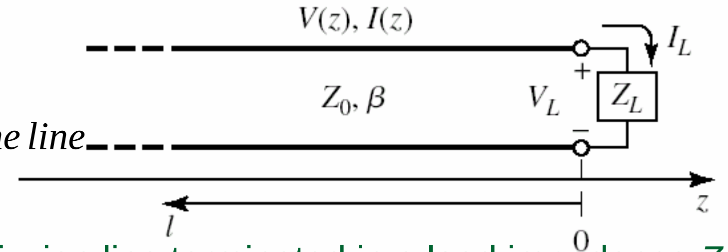
$$==> \quad V_o^- = \frac{Z_L - Z_o}{Z_L + Z_o} V_o^+$$

$$==> \quad \Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$==> \quad V(z) = V_o^+ \left[e^{-j\beta z} + \Gamma e^{j\beta z} \right]$$

total voltage and current waves on the line

$$I(z) = \frac{V_o^+}{Z_o} \left[e^{-j\beta z} - \Gamma e^{j\beta z} \right]$$



A transmission line terminated in a load impedance Z_L.

$$From\ ohmic's\ law: Z_L = \frac{V(0)}{I(0)} = Z_O \cdot \frac{1+\Gamma_O}{1-\Gamma_O}$$

$$==> \Gamma_O = \frac{Z_L - Z_O}{Z_L + Z_O} \quad when\ Z_L = Z_O ==> \Gamma_O = 0$$

$$For\ arbitrary\ of\ z: V(z) = V_O^+ (e^{-\gamma z} + \Gamma_O e^{+\gamma z})\ ;\ I(z) = \frac{V_O^+}{Z_O}(e^{-\gamma z} - \Gamma_O e^{+\gamma z})$$

$$Z_{in}(z) = \frac{V(z)}{I(z)} = Z_O \frac{e^{-\gamma z} + \Gamma_O e^{+\gamma z}}{e^{-\gamma z} - \Gamma_O e^{+\gamma z}} \quad ; because\ \Gamma_O = \frac{Z_L - Z_O}{Z_L + Z_O}$$

$$Z_{in}(z) = Z_O \frac{(Z_L + Z_O)e^{-\gamma z} + (Z_L - Z_O)e^{\gamma z}}{(Z_L + Z_O)e^{-\gamma z} - (Z_L - Z_O)e^{\gamma z}} \quad \left\{ \begin{array}{l} e^{\pm \gamma x} = \cosh(\gamma x) \pm \sinh(\gamma x) \\ \tanh(\gamma x) = \dfrac{\sinh(\gamma x)}{\cosh(\gamma x)} \end{array} \right\}$$

$$Z_{in} = Z_O \frac{Z_L - Z_O \tanh(\gamma z)}{Z_O - Z_L \tanh(\gamma z)}$$

$$==> From\ z = -l, then\ Z_{in}(-l) = Z_O \frac{Z_L + Z_O \tanh(\gamma l)}{Z_O + Z_L \tanh(\gamma l)}$$

$$==> Z_{in}(-l) = Z_O \frac{Z_L + jZ_O \tan(\beta l)}{Z_O + jZ_L \tan(\beta l)} \quad \left(\begin{array}{l} Lossless\ transmission\ line => \gamma = j\beta \\ \tanh(\gamma l) = \tanh(j\beta l) = j\tan(\beta l) \end{array} \right)$$

Reflection coefficient

$$\Gamma(-l) \equiv \frac{V^-(-l)}{V^+(-l)} = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \Gamma_L e^{-j2\beta l} = e^{-j\beta l}\Gamma_L e^{-j\beta l}$$

Voltage standing wave ratio, VSWR

$$VSWR \equiv \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Time-average power flow

$$P_{av}(z) \equiv \frac{1}{T}\int_0^T v(z,t)i(z,t)dt = \frac{1}{2}\mathrm{Re}\left[V(z)I^*(z)\right] = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}\left(1-|\Gamma_L|^2\right)$$

where $P_{av} = \frac{1}{2}\frac{|V_o^+|^2}{Z_o}\mathrm{Re}\left\{1-\Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\right\}$, *middle two terms are purely imaginary* $\Longrightarrow A - A^* = 2j\,\mathrm{Im}(A)$

which shows that the average power flow is constant at any point on the line

→*total power delivered to load is constant = incident power – reflected power*

$\Gamma = 0$ → maximum power is delivered to the load

$|\Gamma| = 1$ → no power is delivered

*When the load is mismatched, not all of the available power from the generator is delivered to the load → Loss is called return loss (**RL**) and is defined in dB*

→$\boldsymbol{RL} = -20\log|\Gamma|\ dB$

If the load is matched to the line → $\Gamma = 0$ *and the magnitude of the voltage on the line is* $|V(z)| = |V_o^+|$ *→ is a constant*

If the load is mismatched→ the presence of a reflected wave leads to standing waves where the magnitude of the voltage on the line is not constant

$$V(z) = V_o^+(e^{-j\beta z} + \Gamma e^{+j\beta z}) = V_o^+ e^{-j\beta z}(1+\Gamma e^{+2j\beta z})$$

$$==> |V(z)| = |V_o^+||1+\Gamma e^{+2j\beta z}| = |V_o^+||1+\Gamma e^{-2j\beta z}| = |V_o^+||1+|\Gamma|e^{j(\theta-2\beta\ell)}|$$

where $\Gamma = |\Gamma|e^{j\theta}$ => $\ell = -z$ is the positive d measured from the load at $z = 0$,

and θ is the phase of the reflection cofficient

$$|V(z)|_{\max} = |V_o^+|(1+|\Gamma|) \text{ when the phase term } e^{j(\theta-2\beta\ell)} = 1 \quad ;$$

$$|V(z)|_{\min} = |V_0^+|(1-|\Gamma_0|) \text{ when the phase term } e^{j(\theta-2\beta\ell)} = -1$$

***Standing Wave Ratio (Voltage Standing Wave Ratio)

As $|\Gamma|$ increases, the ratio of $V_{\max}$ to $V_{\min}$ increases

$$==> VSWR \quad (or\ SWR)\ : VSWR = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

A measure of mismatch of a line

$$==> |\Gamma| = \frac{VSWR-1}{VSWR+1}$$

$RL \equiv -20 \log |\Gamma_L| \quad (dB)$

e.g., $\Gamma_L =$	1	0.1	0
$RL =$	$0\,dB$	$20\,dB$	$\infty\,dB$
VSWR	∞	1.22	1

all incident power reflected — "no return loss"

matched load — "∞ return loss"

$$1 \le VSWR \le \infty$$

matched load $|\Gamma_L| = 0 \rightarrow VSWR = 1$

Impedance match

$Z_{in}(z)=Z_o$ ⇨ no reflected wave $\Gamma(z)=0$, VSWR=1, RL= ∞dB

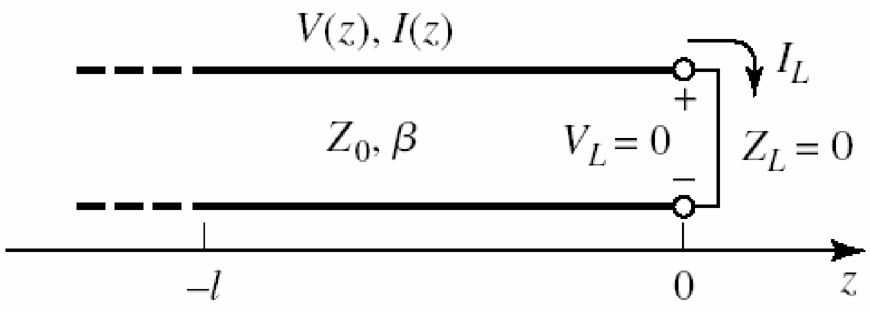
$P_{av}=P_{avmax}$: maximum power delivered to the load

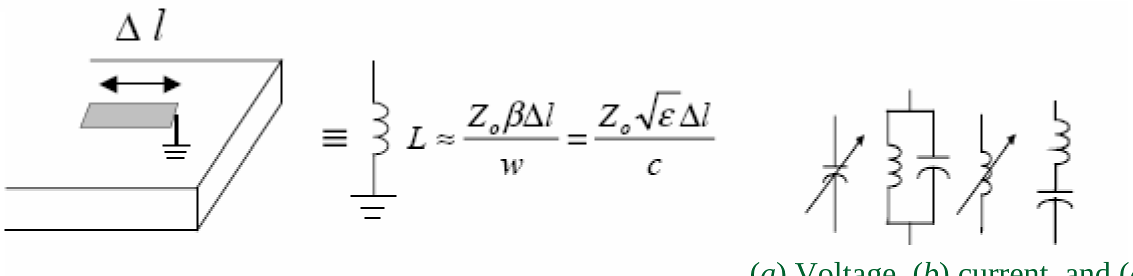
This is an important result giving the input impedance of a length of transmission line with an arbitrary load impedance→ transmission line impedance equation

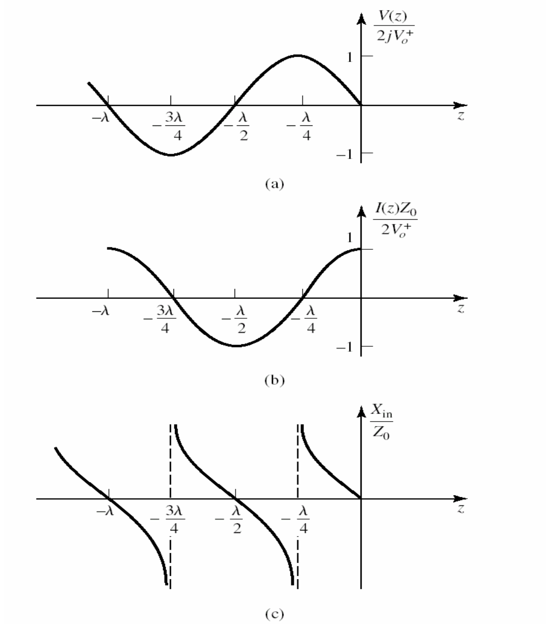
$$Z_{in}(-\ell) = \frac{V(-\ell)}{I(-\ell)} = \frac{V_O^+\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]}{V_O^+\left[e^{j\beta\ell} - \Gamma e^{-j\beta\ell}\right]} Z_O = \frac{1+\Gamma e^{-2j\beta\ell}}{1-\Gamma e^{-2j\beta\ell}} Z_O = Z_O \frac{Z_L + jZ_O \tan(\beta l)}{Z_O + jZ_L \tan(\beta l)}$$

Special Case of Lossless Terminated Lines

For a line is terminated in a short circuit -> $Z_L = 0$ -> $\Gamma = -1$







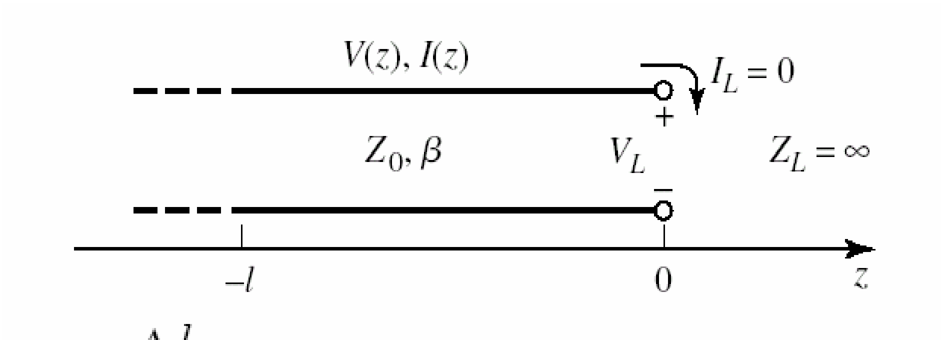
(*a*) Voltage, (*b*) current, and (*c*) impedance (*Rin* = 0 or ∞) variation along a short-circuited transmission line.

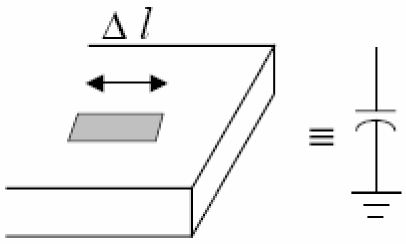
$$V = V_o^+ (e^{-j\beta z} - e^{j\beta z}) = -j2V_o^+ \sin\beta z = j2V_o^+ \sin\beta l \text{ or } \frac{V}{j2V_o^+} = \sin\beta l$$

$$I = \frac{V_o^+}{Z_o}(e^{-j\beta z} + e^{j\beta z}) = 2\frac{V_o^+}{Z_o}\cos\beta z = 2\frac{V_o^+}{Z_o}\cos\beta l \text{ or } \frac{I}{2V_o} = \cos\beta l$$

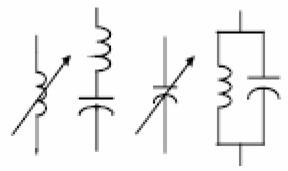
$$Z_{in} = jZ_o \tan\beta l = jX_{in} \text{ or } \frac{X_{in}}{Z_0} = \tan\beta l$$

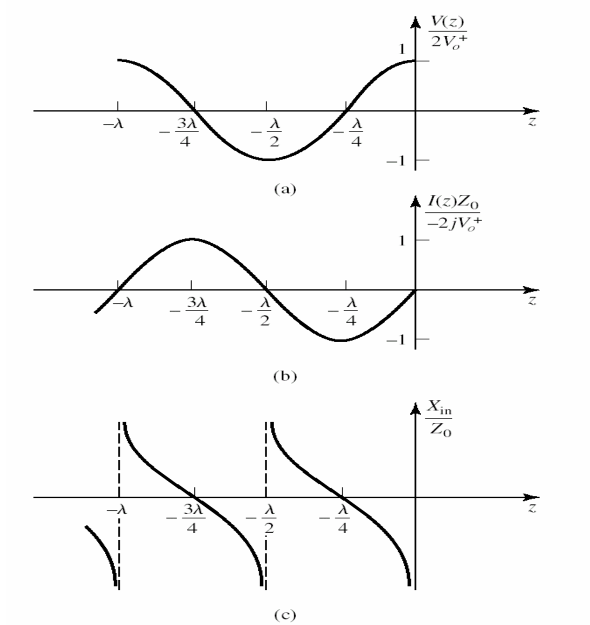
For a line is terminated in a open circuit -> $Z_L = \infty$ -> $\Gamma = 1$





$$C = \frac{\beta \Delta l}{w Z_o} = \frac{\sqrt{\varepsilon}\Delta l}{Z_o c}$$





(*a*) Voltage, (*b*) current, and (*c*) impedance (*Rin* = 0 or ∞) variation along an open-circuited transmission line.

$$V = V_o^+ \left(e^{-j\beta z} + e^{j\beta z}\right) = 2V_o^+ \cos \beta z \quad z < 0$$

$$\because l = -z, V = 2V_o^+ \cos \beta l \;\text{ or }\; \frac{V}{2V_o^+} = \cos \beta l$$

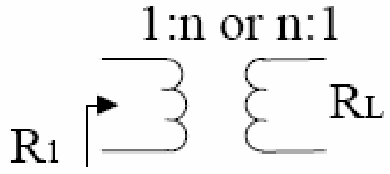
$$I = \frac{V_o^+}{Z_o}\left(e^{-j\beta z} - e^{j\beta z}\right) = -j2\frac{V_o^+}{Z_o}\sin \beta z = j2\frac{V_o^+}{Z_o}\sin \beta l \text{ or } \frac{I}{-j2V_o} = -\sin \beta l$$

$$Z_{in} = \frac{Z_0}{j \tan \beta l} = jX_{in} \text{ or } \frac{X_{in}}{Z_0} = \frac{-1}{\tan \beta l}$$

From consider terminated transmission lines with some special lengths

$l=\lambda/2$, $Z_{in}(l)=Z_L$,

$l=\lambda/4$, $Z_{in}(l)=Z_o^2/Z_L$ quarter-wave "transformer"



$$1:n \rightarrow R_1=\frac{R_L}{n^2},\ n:1 \rightarrow R_1=n^2 R_L$$

$$VSWR=\frac{1+|\Gamma_L|}{1-|\Gamma_L|}=\frac{1+\left|\frac{R_L-Z_o}{R_L+Z_o}\right|}{1-\left|\frac{R_L-Z_o}{R_L+Z_o}\right|}\begin{cases}\overset{R_L>Z_o}{=}\dfrac{R_L}{Z_o},\ R_1=\dfrac{Z_o^2}{R_L}=\dfrac{R_L}{R_L^2/Z_o^2}=\dfrac{R_L}{VSWR^2}\\ \overset{R_L<Z_o}{=}\dfrac{Z_o}{R_L},\ R_1=\dfrac{Z_o^2}{R_L}=\dfrac{R_L}{R_L^2/Z_o^2}=VSWR^2 R_L\end{cases}$$

Consider a transmission line of characteristic impedance Z_o feeding a line of different characteristic impedance Z_1

If the load line is infinitely long, or if it is terminated in its own characteristic impedance, so that there are no reflections from its end, then the input impedance seen by the feed line is Z_1, then the reflection coefficient Γ is

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

A transmission coefficient T

$==>$ *the voltage for* $z < 0$ *is*

$$V(z) = V_o^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right)$$

where V_o^+ is the amplitude of the incident voltage wave on the feed line

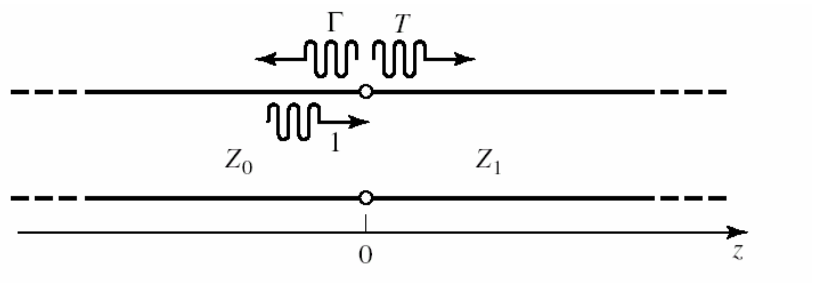
$$V(z) = V_o^+ T e^{-j\beta z} \quad for \quad z > 0$$

Equating these voltage at $z = 0$ gives the transmision coefficient T

$$T = 1 + \Gamma = 1 + \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{2Z_1}{Z_1 + Z_0}$$

IL (insertion loss)

$$IL = -10 \log |T| \quad dB$$



Often the ration of two power levels, P_1 and P_2, in a microwave system is expressed in decibels (dB) as

➔ $10 \log(P_1/P_2)$ dB

→ Using power ratios in dB makes it easy to calculate power loss or gain through a series of components. For ex. : A signal passing through a 6 dB attenuator followed by a 23 dB amplifier will have an overall gain of 23 – 6 = 17 dB.

If $P_1 = V_1^2 / R_1$ and $P_2 = V_2^2 / R_2$, then the resulting power ratio in terms of voltage ratios is

$$10\log\frac{V_1^2 R_2}{V_2^2 R_1} = 20\log\frac{V_1}{V_2}\sqrt{\frac{R_2}{R_1}}\ dB$$

And if the load resistance are equal => 20 log(V_1/V_2) dB

On the other hand, the ratio of voltages across equal load resistances can also be expressed in terms of nepers (Np)

➔ $\ln(V_1/V_2)$ Np → $1/2[\ln(P_1/P_2)]$ Np *since voltage is proportional to the square root of power*

→ $10\ Np = 10 \log e^2 = 8.686\ dB$

If a reference power level is assumed, then absolute powers can also be expressed notation → If we let P_2 = 1mW, then the power P_1 can be expressed in dBm as

$10 \log (P_1/1mW)$ dBm → a power of 1mW is 0dBm, while a power of 1W is 30dBm

Smith Chart

Developed in 1939 by P. Smith at the Bell Tel. Lab. -> impedance matching problem and transmission line issue

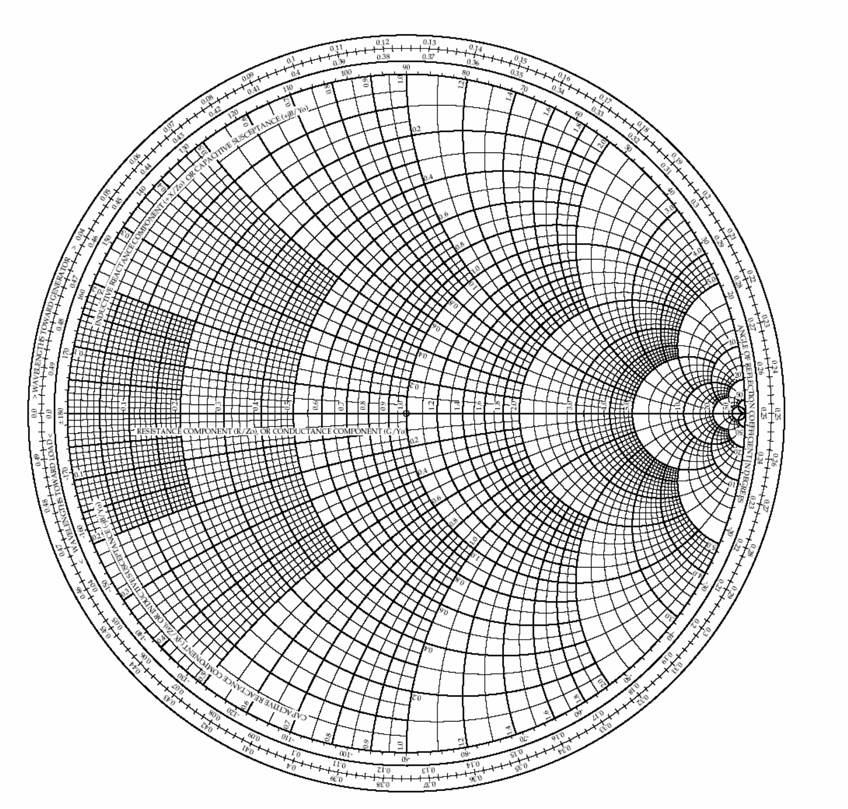
It is essentially a polar plot of the voltage reflection coefficient, Γ

→ *let the reflection coefficient be expressed in magnitude and phase (polar) form as* $\Gamma=|\Gamma|e^{j\theta}$ → *then the magnitude* $|\Gamma|$ *is plotted as a radius* ($|\Gamma|\leq 1$) *from the center of the chart, and the angle* θ ($-180^o \leq \theta \leq 180^o$) *is measured from the right-hand side of the horizontal diameter*

The real utility of the smith chart, it can be used to convert from reflection coefficients to normalized impedances (or admittance)

→*When dealing with impedances on a Smith chart, normalized quantities are generally used* → $z = Z / Z_o$

→ *The normalization constant is usually the characteristic impedance of the line*



If a lossless line of characteristic impedance Z_o is terminated with a load impedance Z_L → $\Gamma = (z_L-1)/(z_L+1) = |\Gamma|\, e^{j\theta}$; where $z_L=Z_L/Z_o$ → this relation can be solved for z_L in terms of Γ to give → $z_L = (1+|\Gamma|e^{j\theta}) / (1-|\Gamma|e^{j\theta})$ where $Z_{in} = [(1+\Gamma e^{-2j\beta l})/(1-\Gamma e^{-2j\beta l})]Z_o$, $l = 0$

This complex equation can be reduced to two real equations by writing Γ and z_L in terms of their real and imaginary parts.

Let $\Gamma = \Gamma_r + j\Gamma_i$ and $z_L = r_L + jx_L$

→ $r_L + jx_L = [(1+\Gamma_r)+j\Gamma_i]/[(1-\Gamma_r)-j\Gamma_i]$

The real and imaginary parts of this equation can be found by multiplying the numerator and denominator by the complex conjugate of the denominator to give

$r_L = [1-\Gamma_r^2-\Gamma_i^2]/[(1-\Gamma_r)^2+\Gamma_i^2]$

$x_L = [2\Gamma_i]/[(1-\Gamma_r)^2+\Gamma_i^2]$

→ $\{\Gamma_r-[r_L/(1+r_L)]\}^2+\Gamma_i^2 = [1/(1+r_L)]^2$ ------- *resistance circles*

$(\Gamma_r-1)^2+[\Gamma_i-(1/x_L)]^2 = (1/x_L)^2$ ------- *reactance circles*

which are seen to represent two families of the circles in the Γ_r and Γ_i

For ex., the $r_L = 1$ circles has its center at $\Gamma_r = 0.5$, $\Gamma_i = 0$ -------- has a radius of 0.5, and so passes through the center of the Smith chart

→ All of the resistance circles have centers on the horizontal $\Gamma_i = 0$ axis, and pass through the $\Gamma = 1$ point on the right-hand side of the chart.

The centers of all the reactance circles lie on the vertical $\Gamma_r =1$ line (off the chart), and these circles also pass through the $\Gamma =1$ point

<u>The resistance and reactance circles are orthogonal.</u>

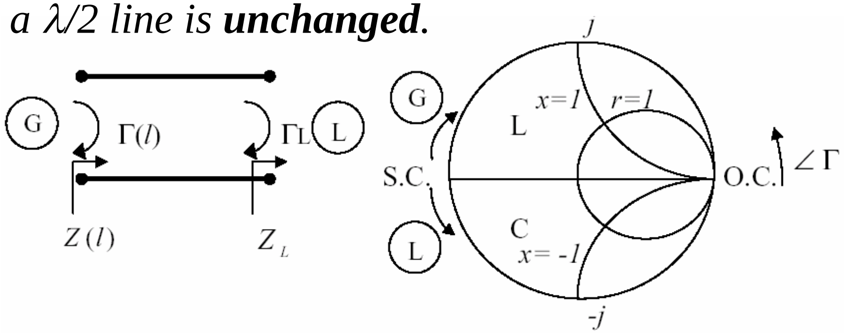
Since terms of the generalized reflection coefficient as

$$Z_{in} = \frac{1+\Gamma e^{-2j\beta\ell}}{1-\Gamma e^{-2j\beta\ell}} Z_O = Z_O \frac{Z_L + jZ_O \tan(\beta l)}{Z_O + jZ_L \tan(\beta l)}$$

where Γ is the reflection at the load, and l is the (positive) length of transmission line. → If we have plotted the reflection coefficient $|\Gamma|e^{j\theta}$ *at the load, the normalized input impedance seen looking into a length l of transmission line terminated with* z_L *can be found by rotating the point clockwise an amount* $2\beta l$ $(\theta - 2\beta l)$ *→ the radius stays the same , since the magnitude of does not change with the position along the line*

The smith chart has scales around its periphery calibrated in the electrical wavelengths, toward and away from the "generator" (the direction away from the load) → these scales are relative, so only the ***difference in the wavelength between two points*** *on the Smith chart* ***is meaningful****.*

=> The ***scales cover a range of 0 to 0.5 wavelengths*** *=>* ***a line of length*** *λ/2 requires a rotation of* $2\beta l = 2\pi$ ***around the center of the chart****,* ***bring the point back to its original position****=> showing that the* ***input impedance*** *of* ***a load*** *seen through a λ/2 line is* ***unchanged****.*



Map rectangular plot of $z = Z/Z_o = r + jx$ on the polar plot of

$\Gamma = |\Gamma|e^{j\angle\Gamma} (= \Gamma_r + j\Gamma_i), |\Gamma| \le 1, -180^o \le \angle\Gamma \le 180^o$

Ex. A load impedance of 40+j70Ω terminated a 100 Ω transmission line that is 0.3λ long. Find the reflection coefficient at the load, the reflection coefficient at the input to the line, the input impedance, the SWR on the line, and the return loss.

<Sol>

The normalized load impedance is $z_L = Z_L / Z_o = 0.4 + j\,0.7$

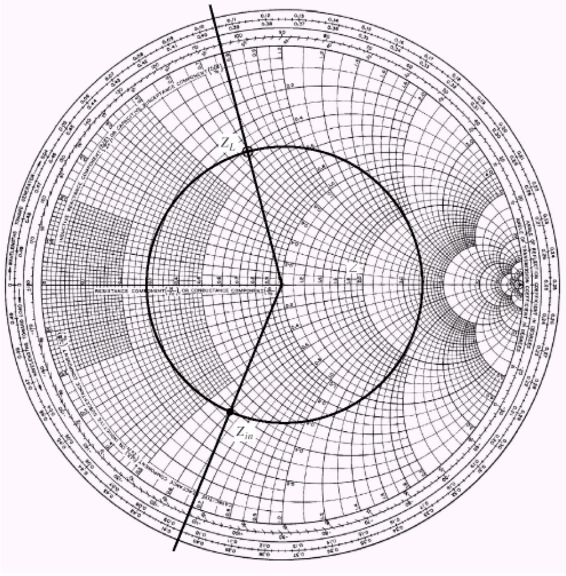
→ *using a compass and the voltage and the voltage coefficient scale below the chart, the reflection coefficient magnitude at the load can be read as* $|\Gamma| = 0.59$ *-> SWR = 3.87, and to the return loss (RL) = 4.6dB* → *Now draw a radial line through the load impedance point, the read the angle of the reflection coefficient at the load from the outer scale of the chart as* 104^o

On the other hand, drawing an SWR circle through the load impedance point.

Reading the reference position of the load on the wavelengths-toward-generator (WTG) scale gives a value of 0.106λ → *moving down the line 0.3λ toward the generator bring to 0.406λ*

→ $Z_{in} = Z_o z_{in} = 100\,(0.365 - j\,0.611) = 36.5 - j\,61.1\Omega$

→ *the reflection coefficient at the input still has a magnitude of* $|\Gamma| = 0.59$ *; phase =* 248^o



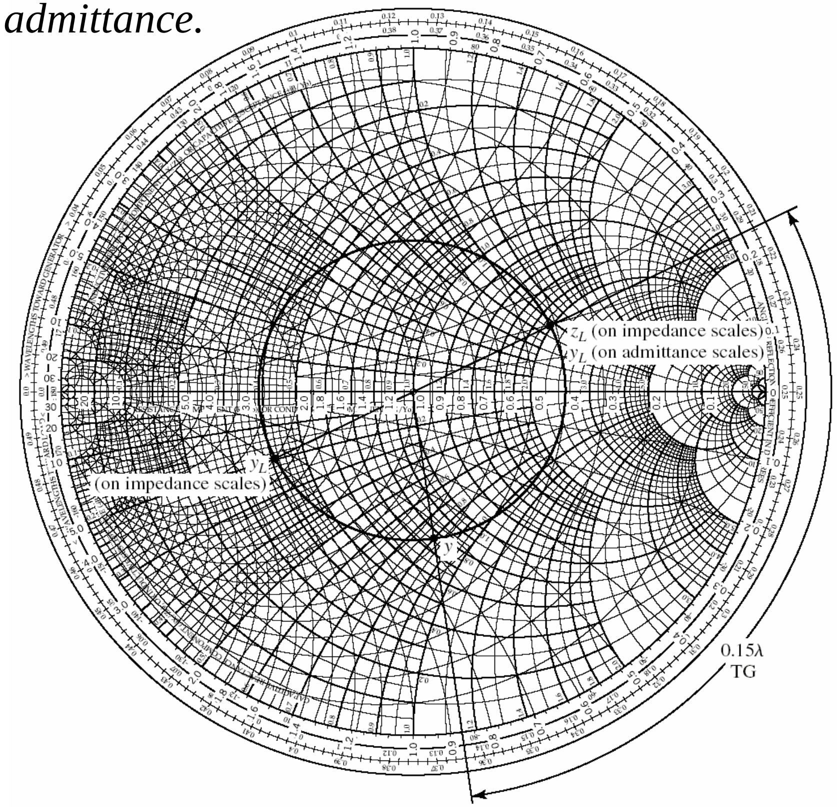
Combined impedance-Admittance Smith Chart

The Smith chart can be used for normalized admittance in the same way that it is used for normalized impedances → it can be used to covert between impedance and admittance

From $Z_{in} = Z_O \dfrac{Z_L + jZ_O \tan(\beta l)}{Z_O + jZ_L \tan(\beta l)}$

the input impedance of load z_L *connected to a* $\lambda/4$ *line is* $z_{in} = 1/z_L$ *which has the effect of converting a normalized impedance to a normalized admittance.*

Since a complete revolution around the Smith chart corresponds to a length of $\lambda/2$, *a* $\lambda/4$ *transformation is equivalent to rotating the chart by* 180^o *; this is also equivalent to imaging a given impedance (or admittance) point across the center of the chart to obtain the corresponding admittance (or impedance) point.*



Ex. Smith Chart Operations Using Admittances

A load of $Z_L = 100 + j\,50\Omega$ terminated a 50Ω line. What are the load admittance and the input admittance if the line is 0.15λ long ?

<Sol>

Normalized load impedance is $z_L = 2 + j\,1$ → plotted the z_L and SWR circle

→*Conversion to admittance can be accomplished with a $\lambda/4$ rotation of z_L (or drawing a straight line through z_L and the center of the chart to intersect the SWR circle) ; The chart can now be considered as an admittance chart, and the input impedance can be rotating 0.15λ from y_L.*

Plotting zL on the impedances scales and reading the admittance scales at this same give $y_L = 0.4 - j\,0.2$ => the actual load admittance is then

$$Y_L = y_L Y_o = y_L / Z_o = 0.008 - j\,0.004\ S$$

Then , on the WTG scale, the load admittance is seen to have a reference position of 0.214λ. Moving 0.15λ → 0.364λ

⇒*A radial line at this point on the WTG scale intersects the SWR circle at* ***an admittance of $y = 0.61 + j\,0.66$***

⇒→ ***actual input admittance is then $Y = 0.0122 + j\,0.0132\ S$***

Slotted Line

A slotted line is a transmission line configuration (usually waveguide or coax) that allows the sampling of the electrical field amplitude of a standing wave on a terminated line. → with this device the **SWR and the distance of the first voltage minimum from the load can be measured**, and **from this data the load impedance can be determined** → due to the load impedance is in general a complex number (with two degrees of freedom), **two distinct quantities must be measured with the slotted line to uniquely determine this impedance**

→ Measured impedance

Slotted Line (previous) → Vector Network Analyzer (now)

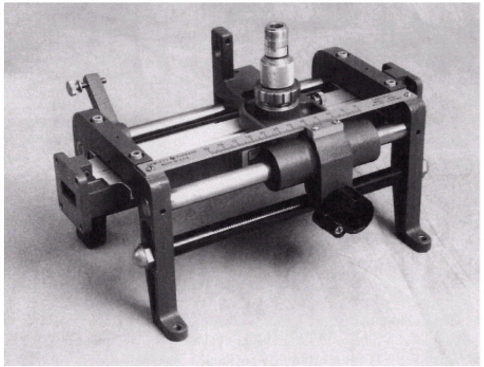
Assume that, for a certain terminated line, we have measured the SWR on the line and l_{min}, the distance from the load to the first voltage minimum on the line. The load impedance Z_L can be determined as follows.

$|\Gamma| = (SWR-1)/(SWR+1)$ *; a voltage minimum occurs when* $e^{j(\theta - 2\beta l)} = -1$, *when* θ *is the phase angle of the reflection coefficient,* $\Gamma = |\Gamma| e^{j\theta}$

=> $\theta = \pi + 2\beta l_{min}$ *where* l_{min} *is the distance from the load to the first voltage minimum*

Since the voltage minimums repeat every $\lambda/2$, *where* λ *is the wavelength on the line, and multiple of* $\lambda/2$ *can be added to* l_{min} *without changing the result in* $\theta = \pi + 2\beta l_{min}$, *because this just amounts to adding* $2\beta n\lambda/2 = 2\pi n$ *to* θ, *which not change* Γ
→ *the complex reflection coefficient* Γ *at the load can be find by SWR and* l_{min}

To find the load impedance form Γ *with* $l = 0$: $Z_L = Z_o [(1+\Gamma)/(1-\Gamma)]$



An X-band waveguide slotted line.

The Quarter-Wave Transformer

The quarter-wave transformer is a useful and practical circuit for impedance matching and also provides a simple transmission line circuit that further illustrates the properties of standing waves on a mismatched line.

For Impedance Viewpoint

These two components are connected with a lossless piece of transmission line of characteristic impedance Z_1 and length $\lambda/4$ → It is desired to match the load to the Z_o line, by using the $\lambda/4$ piece of line, and so make $\Gamma = 0$ looking into the $\lambda/4$ matching section.

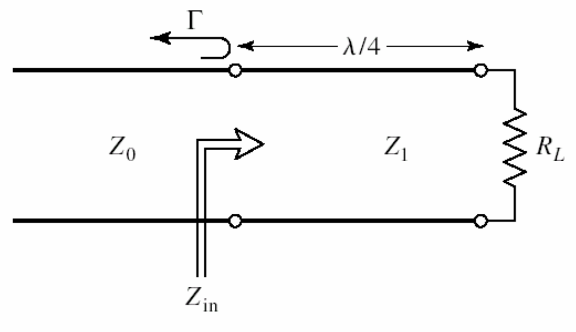
$\Rightarrow$ $$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan \beta\ell}{Z_1 + jR_L \tan \beta\ell}$$ => to evaluate this for $\beta l = (2\pi\lambda)(\lambda/4) = \pi/2$

$\Rightarrow$ we can divide the numerator and denominator by tan βl and take the limit as $\beta l \rightarrow \pi/2$ to get

$$Z_{in} = Z_1^2 / R_L$$

In order for $\Gamma = 0$, we must have $Z_{in} = Z_o$, which yields the characteristic impedance Z_1 as

$$Z_1 = \sqrt{Z_o R_L}$$ the geometric mean of the load and source impedances



When the length of the matching section is $\lambda/4$,
or an odd multiple (2n+1) of $\lambda/4$ long,
so that a perfect match may be achieved at one frequency,
but mismatch will occur at other frequencies.

Ex. Consider a load resistance R_L = 100Ω, to be matched to a 50Ω line with a quarter-wave transformer. Find the characteristic impedance of the matching section and plot the magnitude of the reflection coefficient versus normalized frequency, f/f_o, where f_o is the frequency at which the line is λ/4 long.

<Sol>

$$z_1 = \sqrt{(50)(100)} = 70.71\Omega$$

The reflection coefficient magnitude is given as

$$|\Gamma| = \left| \frac{Z_{in} - Z_o}{Z_{in} + Z_o} \right|$$

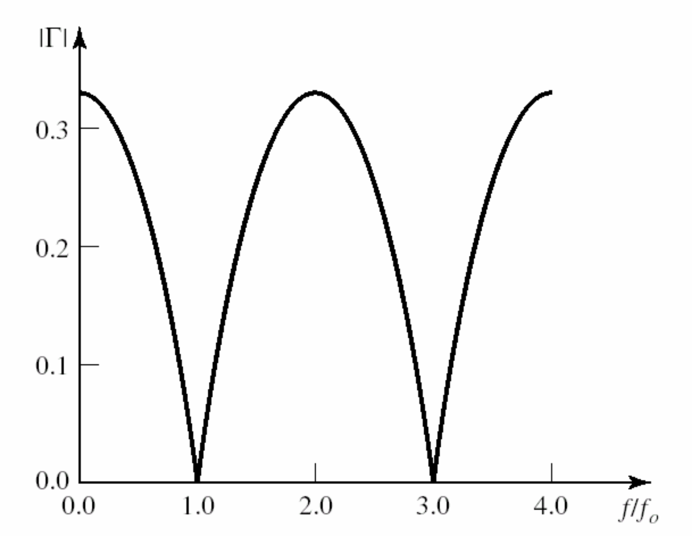
where the input impedance Z_{in} *is a function of frequency*

The frequency dependence in comes from the $\beta\ell$ *term, which can be written in terms of* f/f_o *as*

$$\beta\ell = \left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda_o}{4}\right) = \left(\frac{2\pi f}{v_p}\right)\left(\frac{v_p}{4f_o}\right) = \frac{\pi f}{2f_o}$$

For higher frequencies the line looks electrically longer, and for lower frequencies it looks shorter.

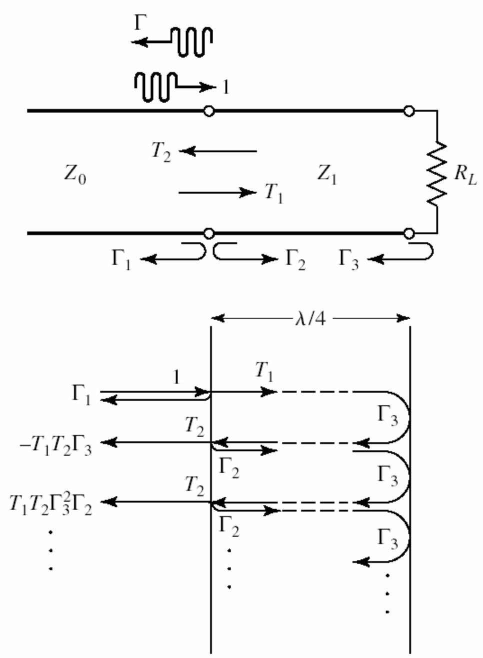
The magnitude of the reflection coefficient is plotted versus f/f_o



The Multiple Reflection Viewpoint

Γ = overall, or total, reflection coefficient of a wave incident on the $\lambda/4$ transformer
Γ_1 = partial reflection coefficient of a wave incident on a load Z_1, from the Z_o line
Γ_2 = partial reflection coefficient of a wave incident on a load Z_o, from the Z_1 line
Γ_3 = partial reflection coefficient of a wave incident on a load R_L, from the Z_1 line
T_1 = partial transmission coefficient of a wave from the Z_0 line into the Z_1 line
T_2 = partial transmission coefficient of a wave from the Z_1 line into the Z_0 line

$\Gamma_1 = (Z_1-Z_o) / (Z_1+Z_o)$
$\Gamma_2 = (Zo-Z_1) / (Z_o+Z_1) = -\Gamma_1$
$\Gamma_3 = (R_L-Z_1) / (R_L+Z_1)$
$T_1 = 2Z_1 / (Z_1+Z_o)$
$T_2 = 2Z_o / (Z_1+Z_o)$



Clearly, this process continues with an infinite number of bouncing waves, And the total reflection coefficient is the sum of all of these partial reflections. Since each round trip path up and down the $\lambda/4$ transformer Section results in a 180° phase shift, the total reflection coefficient can be expressed as

$$\Gamma = \Gamma_1 - T_1T_2\Gamma_3 + T_1T_2\Gamma_2\Gamma_3^2 - T_1T_2\Gamma_2^2\Gamma_3^3 + \ldots\ldots$$

$$= \Gamma_1 - T_1T_2\Gamma_3\sum_{n=0}^{\infty}\left(-\Gamma_2\Gamma_3\right)^n$$

$when\ |\Gamma_3| < 1\ \ and\ \ |\Gamma_2| < 1$, the infinte series can be using the geometric series result that

$$\sum_{n=0}^{\infty} x^n = \frac{1}{x-1}\ \ for\ |x| < 1$$

to give

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3} = \frac{\Gamma_1 + \Gamma_1\Gamma_2\Gamma_3 - T_1 T_2 \Gamma_3}{1+\Gamma_2\Gamma_3}$$

The number of this expression can be simplified to give

$$\Gamma_1 - \Gamma_3\left(\Gamma_1^2 + T_1 T_2\right) = \Gamma_1 - \Gamma_3\left[\frac{(Z_1 - Z_o)^2}{(Z_1 + Z_o)^2} + \frac{4 Z_1 Z_o}{(Z_1 + Z_o)^2}\right] = \Gamma_1 - \Gamma_3$$

$$= \frac{(Z_1 - Z_o)(R_L + Z_1) - (R_L - Z_1)(Z_1 + Z_o)}{(Z_1 + Z_o)(R_L + Z_1)} = \frac{2\left(Z_1^2 - Z_o R_L\right)}{(Z_1 + Z_o)(R_L + Z_1)}$$

which is seen to vanish if we choose $\mathrm{Z}_1 = \sqrt{Z_o R_L}$

Then Γ is zero, and the line is matched

Generator and Load Mismatches

In general, both generator and load may present ***mismatched impedances to the transmission line****. We will study this case, and also see that the condition for* ***the maximum power transfer from the generator to the load****, in some situations, require a standing wave on line.*

→ Figure shows a transmission line circuit with arbitrary generator and load impedance, Z_g and Z_l, which may be complex.→ transmission line is assumed lossless with a length l and characteristic impedance Z_o => Due to mismatched → multiple reflections can occur on the line → problem of the quarter-wave transformer

The input impedance looking into the terminated transmission line from the generator end is

$$Z_{in} = Z_o \frac{1+\Gamma_\ell e^{-2j\beta\ell}}{1-\Gamma_\ell e^{-2j\beta\ell}} = Z_o \frac{Z_\ell + jZ_o \tan\beta\ell}{Z_o + jZ_\ell \tan\beta\ell}$$

where Γ_ℓ is the reflection coefficient of the load $\Gamma_\ell = \frac{Z_\ell - Z_o}{Z_\ell + Z_o}$

The voltage on the line can be written as $V(z) = V_o^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$

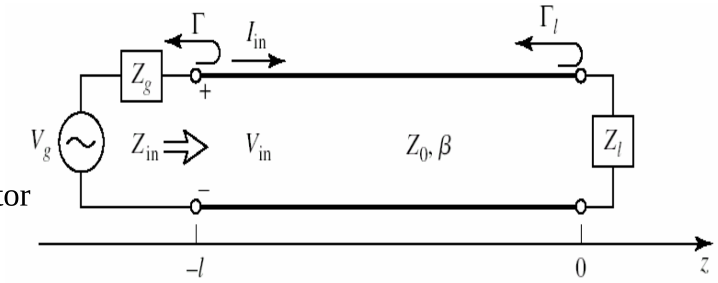
and we can find V_o^+ from the voltage at the generator end of the line, where $z = -\ell$

$$\Rightarrow \quad V(-\ell) = V_g \frac{Z_{in}}{Z_{in}+Z_g} = V_o^+\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)$$

$$\Rightarrow \quad V_o^+ = V_g \frac{Z_{in}}{Z_{in}+Z_g} \frac{1}{\left(e^{j\beta\ell} + \Gamma_\ell e^{-j\beta\ell}\right)}$$

$$\Rightarrow \quad V_o^+ = V_g \frac{Z_o}{Z_o+Z_g} \frac{e^{-j\beta\ell}}{\left(1-\Gamma_\ell \Gamma_g e^{-2j\beta\ell}\right)}$$

where Γ_g is the reflection coefficient seen looking into the generator



$$\Rightarrow \text{SWR} = \frac{1+|\Gamma_\ell|}{1-|\Gamma_\ell|}$$

The power delivered to the load is

$$P=\frac{1}{2}\mathrm{Re}\{V_{in}I_{in}^{*}\}=\frac{1}{2}|V_{in}|^2\,\mathrm{Re}\left\{\frac{1}{Z_{in}}\right\}=\frac{1}{2}|V_g|^2\left|\frac{Z_{in}}{Z_{in}+Z_g}\right|^2\mathrm{Re}\left\{\frac{1}{Z_{in}}\right\}$$

Now let $Z_{in}=R_{in}+jX_{in}$ and $Z_g=R_g+jX_g$

$$\Rightarrow P=\frac{1}{2}|V_g|^2\frac{R_{in}}{(R_{in}+R_g)^2+(X_{in}+X_g)^2}$$

Load Matched to Line

=> $Z_l = Z_o$ → $\Gamma_\ell = 0$ *and SWR = 1 => the input impedance is* $Z_{in} = Z_o$

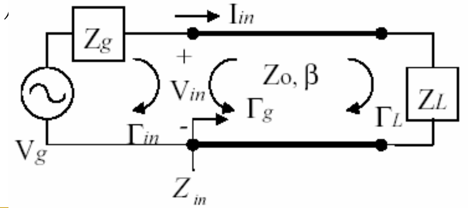
=> *the power delivered to the load is* $P=\frac{1}{2}|V_g|^2\frac{Z_o}{(Z_o+R_g)^2+X_g^2}$

Generator Matched to Loaded Line

The load impedance Z_ℓ *and/or the transmission line parameters* $\beta\ell$, Z_o *are chosen to make the input impedance* $Z_{in} = Z_g$, *so that the generator is matched to the load presented by the terminated transmission line => the overall reflection coefficient,* Γ, *is zero =>* $\Gamma=(Z_{in}-Z_g)/(Z_{in}+Z_g)=0$

→ *However, a standing wave on the line since* Γ_ℓ *may not be zero*

The power delivered to the load is $P=\frac{1}{2}|V_g|^2\frac{R_g}{4(R_g^2+X_g^2)}$



$$P_{in}=\frac{1}{2}Re(V_{in}I_{in}^{*})=\frac{1}{2}|V_g|^2\frac{R_{in}}{(R_{in}+R_g)^2+(X_{in}+X_g)^2}$$

Conjugate Matching

Assuming that the generator series impedance, Z_g, is fixed, we may vary the input impedance Z_{in} until we achieve the maximum power delivered to the load.

=> Knowing Z_{in} → easy to find Z_ℓ via an impedance transformation along the line

To maximum P, we differentiate with respect to the real and imaginary parts of Z_{in}

$$\frac{\partial P}{\partial R_{in}} = 0 \rightarrow \frac{1}{(R_{in}+R_g)^2+(X_{in}+X_g)^2} + \frac{-2R_{in}(R_{in}+R_g)}{\left[(R_{in}+R_g)^2+(X_{in}+X_g)^2\right]^2} = 0$$

$$or, \qquad R_g^2 - R_{in}^2 + (X_{in}+X_g)^2 = 0$$

$$\frac{\partial P}{\partial X_{in}} = 0 \rightarrow \frac{-2R_{in}(X_{in}+X_g)}{\left[(R_{in}+R_g)^2+(X_{in}+X_g)^2\right]^2} = 0$$

$$or, \qquad X_{in}(X_{in}+X_g) = 0$$

solving simulatenously for R_{in} and X_{in} gives

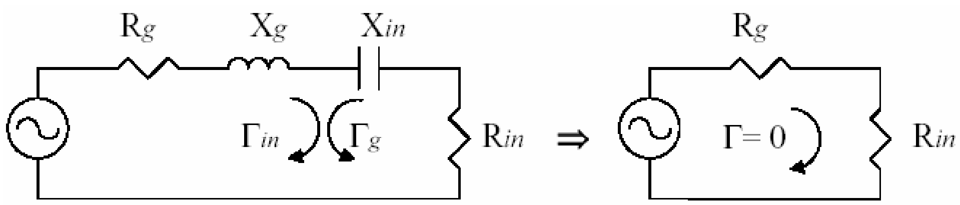
$$R_{in} = R_g, \qquad X_{in} = -X_g \qquad (\Gamma_g \neq 0, \Gamma_{in} \neq 0) \Rightarrow \text{maximum power transfer}$$

$$or, \qquad Z_{in} = Z_g^*$$

This condition is known as conjugate matching, and results in maximum power transfer to the load, for a fixed generator impedance

The power delivered is

$$P = \frac{1}{2}|V_g|^2 \frac{1}{4R_g}$$



Lossy Transmission Lines

In practice, all transmission lines have loss due to finite conductivity and/or lossy dielectric. → we will study the effect of loss on transmission line behavior and show how the attenuation constant can be calculated.

$$For\;\; low\;\; loss\;\; line \Rightarrow R << \omega L \;\; G << \omega C$$

The general experssion for he complex propagation constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(j\omega L)(j\omega C)\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right) - \frac{RG}{\omega^2 LC}}$$

with $RG << \omega^2 LC$

$$\gamma = j\omega\sqrt{LC}\sqrt{1 - j\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)}$$

If we were to ignore the $(R/\omega/ \; + G/\omega/ \;)$ and using Taylor series expression

$$\gamma \cong j\omega\sqrt{LC}\left[1 - \frac{j}{2}\left(\frac{R}{\omega L} + \frac{G}{\omega C}\right)\right], \;\; \text{so that} \;\; \alpha \cong \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(\frac{R}{Z_o} + GZ_o\right)$$

$$\beta \cong \omega\sqrt{LC}$$

$$\Rightarrow Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \cong \sqrt{\frac{L}{C}}$$

- distortionless line RC=LG

$$\alpha = R\sqrt{\frac{C}{L}} : \text{constant}, \beta = w\sqrt{LC} \rightarrow v_p = \frac{1}{\sqrt{LC}} : \text{constant}, \Delta t = \frac{\Delta l}{v_p} : \text{constant}$$

$$Z_o = \sqrt{\frac{L}{C}}$$

- perturbation method

low-loss line (assume $\Gamma(z)=0$) *where P_o is the power at the z=0 plane*

$$P(z) = P_o e^{-2\alpha z} \rightarrow \text{power loss/length } P_l \equiv -\frac{\partial P}{\partial z} = 2\alpha P(z)$$

$$\Rightarrow \alpha = \frac{P_l(z)}{2P(z)} = \frac{P_l(z=0)}{2P_o}$$

The Terminated Lossy Line

$V(z) = V_o^+\left[e^{-\gamma z} + \Gamma e^{\gamma z}\right]$

$I(z) = \dfrac{V_o^+}{Z_o}\left[e^{-\gamma z} - \Gamma e^{\gamma z}\right]$ where Γ is the reflection coefficient of the load and

V_o^+ is the incident voltage amplitude reference at z = 0

$\Rightarrow \Gamma(\ell) = \Gamma e^{-2j\beta\ell} e^{-2\alpha\ell} = \Gamma e^{-2\gamma\ell}$ the reflection coefficient at a distance ℓ from the load

The input impedance Z_{in} at a distance ℓ from the load

$$\Rightarrow Z_{in} = \frac{V(-\ell)}{I(-\ell)} = Z_o \frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}$$

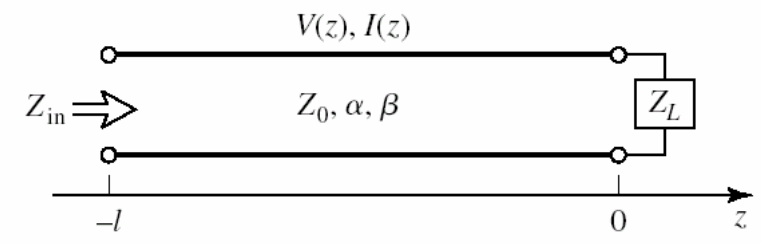
$\Rightarrow$ the power delivered to the input of the terminated line at z = -ℓ as

$$P_{in} = \frac{1}{2}\mathrm{Re}\{V(-\ell)I(-\ell)\} = \frac{|V_o^+|^2}{2Z_o}\left[e^{2\alpha\ell} - |\Gamma|^2 e^{-2\alpha\ell}\right] = \frac{|V_o^+|^2}{2Z_o}\left[1 - |\Gamma(\ell)|^2\right]e^{2\alpha\ell}$$

The power actually delivered to the load is

$$P_L = \frac{1}{2}\mathrm{Re}\{V(0)I^*(0)\} = \frac{|V_o^+|^2}{2Z_o}\left(1 - |\Gamma|^2\right)$$

The difference in thee powers corresponds to the power lost in the line



$$P_{loss} = P_{in} - P_L = \frac{|V_o^+|^2}{2Z_o}\left[\left(e^{2\alpha\ell} - 1\right) + |\Gamma|^2\left(1 - e^{-2\alpha\ell}\right)\right]$$

$\Rightarrow$ The first term accounts for the power loss of the incident wave,

while the second term accounts for the power loss of the reflected wave

note : that both terms increase as α increases